Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

APPLICATION

OF NAPIER'S RULES FOR SOLVING THE CASES OF RIGHT-ANGLED SPHERIC TRIGONOMETRY TO SEVERAL CASES OF OBLIQUE-ANGLED SPHERIC TRIGONOMETRY.

By **NATHANIEL BOWDITCH**, A. M. et A. A. S.

BY the celebrated rules of Lord Napier the whole of Right-Angled Spheric Trigonometry is reduced to two simple analogies or equations, which, being very easy to be remembered, are much made use of by mathematicians. The object of the present memoir is to point out a small alteration in the expression of those rules, so that they may include the solutions of most of the cases of Oblique-Angled Spheric Trigonometry in a more simple manner than in their original form.

In every Right-Angled Spheric Triangle there are five circular parts ; namely, the two legs, the complement of the hypotenuse, and the complements of the two oblique angles, which are named *adjacent* or *opposite*, according to their positions with respect to each other. The right angle is not included as one of the circular parts, neither is it supposed to separate the legs. In all cases of Right-Angled Spheric Trigonometry two of these parts are given to find a third. *If the three parts join, that, which is in the middle, is called the middle part : if they do not join, two of them must, and the other part, which is separate, is called the middle part, and the other two, opposite parts, as in Fig.* 1, 2. Then, putting the radius equal to unity, the equations given by Napier will become

Sine of middle part = Rectangle of the tangents of the adjacent parts.
= Rectangle of the cosines of the opposite parts.

The method of applying these solutions to the various cases of Right-Angled Spheric Trigonometry is very simple, and is explained in several treatises of Trigonometry.

To apply this method to Oblique-Angled Spheric Trigonometry it is necessary to divide the triangle into two right-angled spheric triangles by means of a perpendicular AP (Fig. 3) let fall from the point A upon the opposite side BC : *the perpendicular being so chosen as to make two of the given things fall in one of the right-angled triangles.** Each triangle thus formed contains, as above, five circular parts, the perpendicular being counted and bearing the same name in each of them: consequently the parts of each triangle similarly situated with respect to the perpendicular must have the same name. In every case of Oblique-Angled Spheric Trigonometry there are three parts given to find a fourth, and in most of the cases there are two of these parts in each of the triangles ACP, ABP, similarly situated with respect to each other. To each of these must be joined the perpendicular AP, and there will be three parts in each triangle, which are to be named *middle*, *adjacent*, or *opposite*, according to the above directions. Then the equations for solving all the cases of Right-Angled, and most of the cases of Oblique-Angled Spheric Trigonometry† are

$$\text{Sine middle part}\left\{\begin{matrix} = \\ \propto \end{matrix}\right\}\begin{matrix}\text{Tangents of the adjacent parts.}\\ \text{Cosines of the opposite parts.}\end{matrix}$$

These equations, when applied to Right-Angled Spheric Triangles, signify, as before, that the sine of the middle part is equal to the rectangle of the tangents of the adjacent parts, or to the rectangle of the cosines of the opposite parts ; but when applied to an Oblique-Angled Triangle, they signify, that the sines of the middle parts are respectively proportional to the tangents of the adjacent parts, or to the cosines of the opposite parts of the same triangle ; observing, that the perpendicular being common to both triangles APB, APC, and having the same name in each of them cannot be made use of in the analogies, and must not therefore be counted as a middle part. This circumstance can produce no embarrassment, because all the cases in Oblique Spheric Trigonometry may be solved without calculating the length of the perpendicular.

---

* Or in other words the perpendicular ought to be let fall from the end of a given side and opposite to a given angle. When this can be done in two different ways, that one is to be rejected, which makes the perpendicular a middle part. Thus in Example 3 the perpendicular ought not to be let fall from the point B.

† The cases excepted are where three sides or three angles are given ; or where the question relates to two angles and their opposite sides. It may be proper to observe, that it will be of considerable assistance in remembering the above rules to take notice of the circumstance, that the second letters of the words *tangent* and *cosine* are the same as the first letters of *adjacent* and *opposite*.

These rules for solving the cases of Oblique Spheric Trigonometry are easily deduced from those given by Lord Napier. For if we put M for the middle part, A for the adjacent part, and B for the opposite part of the triangle APC (Fig. 3, 4, or 5), *m*, *a*, *b*, for the corresponding parts of the triangle APB, and P for the perpendicular AP. Then, if P is an adjacent part, we shall have by Napier's rules, tang. P$=\frac{\text{sine M}}{\text{tang. A}}$, and tang. P$=\frac{\text{Sine } m}{\text{tang. } a}$, hence $\frac{\text{Sine M}}{\text{tang. A}}=\frac{\text{Sine } m}{\text{tang. } a}$, consequently Sine M : tang. A :: sine *m* : tang. *a* ; and if P is an opposite part, we shall have, cos. P$=\frac{\text{Sine M}}{\text{Cos. B}}$, and cos. P$=\frac{\text{sine } m}{\text{cos. } b}$. Hence $\frac{\text{sine M}}{.}=\frac{\text{sine } m}{\text{cos. } b}$, consequently sine M : cos. B :: sine *m* : cos. *b*. Which are the two propositions to be demonstrated.

To illustrate these rules the following examples are given.

EXAMPLE 1.

Given AB, AC, and the angle C, to find BC, Fig. 3.

In the right-angled triangle ACP we have AC and C ; whence we may find PC by the rule, sine of middle part = tangents adjacent parts, which gives (as in Fig. 2) sine (co. C) = tang. PC × tang. (co. AC), whence tang. PC = cos. C × tang. AC. Then in the triangles ABP, APC, we have AB, AC and PC to find BP, and if to these we join the perpendicular AP, we shall find, in the triangle APC, that the complement of AC is the middle part and PC an opposite part. The triangle ABP is to be marked in a similar manner. Then the rule, Sine middle part $\propto$ cosines of the opposite parts, gives Sine (co. AC) : cos. PC : : sine (co. AB) : cos. BP. Having found BP we have BC $= \mathrm{BP} \underset{\backsim}{+} \mathrm{PC}$.

EXAMPLE 2.

Given BC, AC, and the angle C, to find AB. Fig. 3.

As in the last example we find PC, thence $\mathrm{BP} = \mathrm{BC} \underset{\backsim}{+} \mathrm{PC}$. Then in the triangles APC, ABP we have AC, PC and PB to find AB, which requires that the triangles should be marked as in the last example, and the rule

Sine middle part $\propto$ cosines of opposite parts, gives sine (co. AC) : cos. PC : : sine (co. AB) : cos. BP, whence we obtain cos. PC : cos. AC : : cos. BP : cos. AB.

EXAMPLE. 3.

Given AC, BC and the angle C to find the angle B. Fig. 4.

Find PC, PB, as in the preceding example, then the rule, sine middle part $\propto$ tangents of adjacent parts gives sine PC : tang. (co. C) : : sine BP : tang. (co. B).

EXAMPLE 4.

Given the angles A and C and the side AC to find AB. Fig. 5.

In the right angled triangle ACP we have AC and the angle C, whence we may find the angle PAC by the rule, Sine middle part = tangents adjacent parts, which gives, (as in fig. 1.) Sine (co. AC)

= tang. (co. C) × tang. (co. PAC), or cotang. PAC = cos. AC × tang. C, whence we have BAP = BAC $\underset{\backsim}{+}$ PAC. Then in the triangles ABP, APC we have the angles PAC, BAP, and the side AC, to find AB; and if to these we join the perpendicular AP we shall find, in the triangle APC, that the complement of the angle PAC is the middle part and the complement of AC is an adjacent part; and the triangle APB is to be marked in a similar manner. Whence the rule, Sine middle part ∝ tangents adjacent parts, gives sine (co. PAC) : tang. (co. AC) : : sine (co. PAB) : tang. (co. AB).

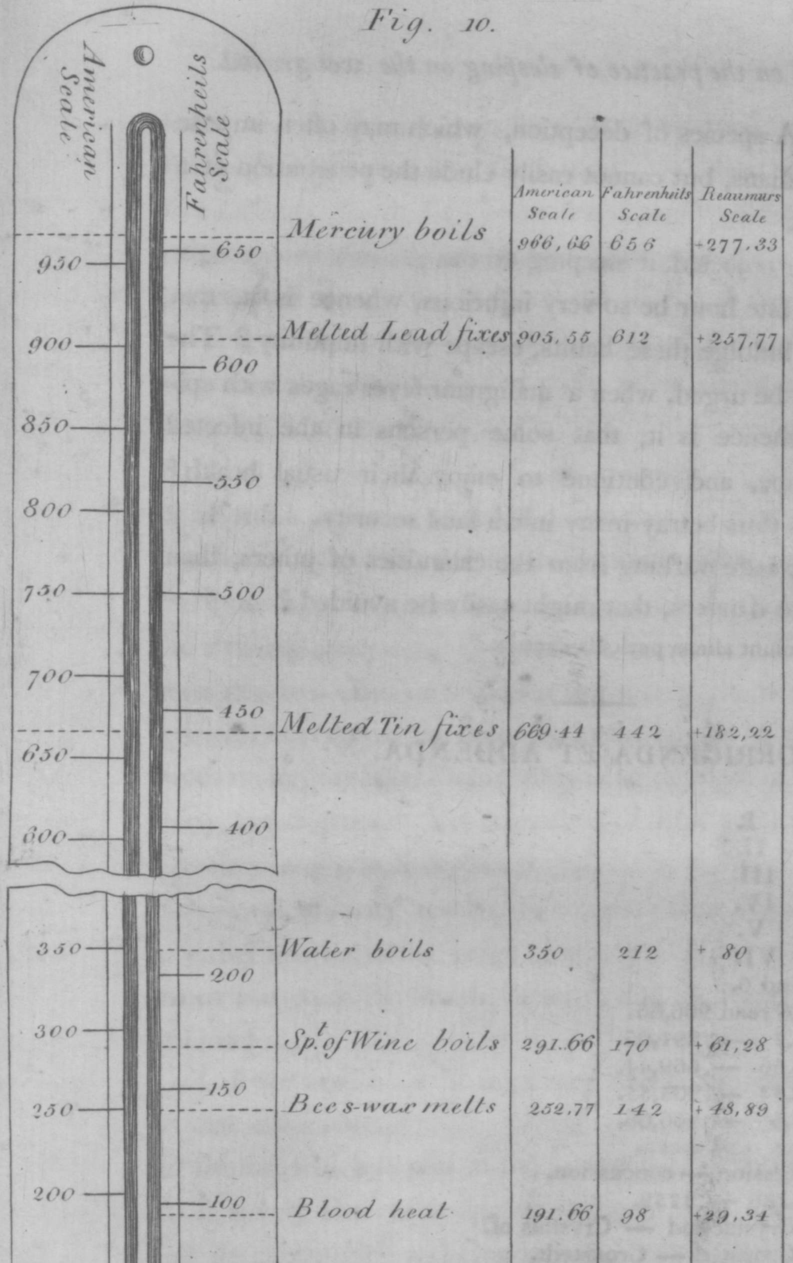
In the same manner we may proceed with any other examples; taking care always to exclude the perpendicular from the analogies.

This method embraces all the cases of Spheric Trigonometry excepting two; namely, where the question is between two sides and the opposite angles of an oblique angled triangle, or where three sides are given to find an angle. The first of these may be solved by the noted rule, that the sine of a side is proportional to the sine of its opposite angle. Thus if AB, AC, and B were given to find C, we should have sine AC : sine B : : sine AB : sine C. The case where three sides are given includes that where three angles are given, by taking the supplementary triangle. The usual rule for finding one of the angles as B, when the three sides AB, AC, BC are given, is by putting $s = \frac{1}{2}\,(AB + AC + BC)$ then

$$\text{Log.}\,\text{cos.}\,\tfrac{1}{2}\,B = \tfrac{1}{2} \times \left\{ \text{log.}\,\text{sine}\; s + \text{log.}\,\text{sine}\; \overline{s - AC} + \text{log.}\,\text{cosec.}\,AB + \text{log.}\,\text{cosec.}\,BC \right\}$$

Both these excepted cases occur so frequently in practice that it becomes quite easy to remember the rules for solving them, and if to these we join the rules of Napier, altered in the manner we have here suggested, it will be very easy to remember all the rules necessary for solving the various cases of right-angled and oblique-angled Spheric Trigonometry.

Fig. 10.



	American Scale	Fahrenheits Scale	Reaumurs Scale
Mercury boils	966, 66	656	+277.33
Melted Lead fixes	905.55	612	+257.77
Melted Tin fixes	669.44	442	+182,22
Water boils	350	212	+80
Sp^{t}. of Wine boils	291.66	170	+61,28
Bees-wax melts	252.77	142	+48,89
Blood heat	191.66	98	+29.34
Water freezes	100	32	0
Zero of Fahrenheit	55.55	0	−14.2
Mercury freezes	0	−40	−32

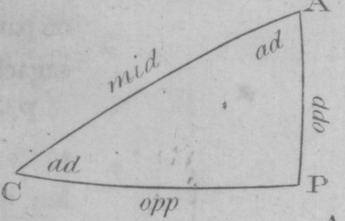


Fig. 1.

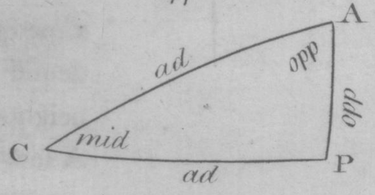


Fig. 2.

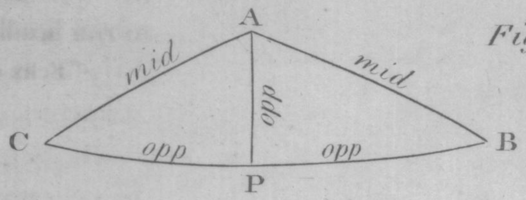


Fig. 3.

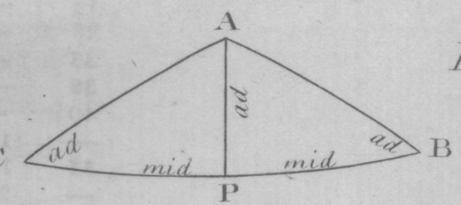


Fig. 4.

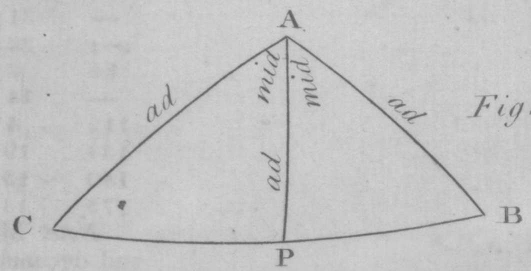


Fig. 5

Wighlman Sc.